LATENT TRAIT MODELLING: FUNDAMENTAL MEASUREMENT IN THE HUMAN SCIENCES

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ABSTRACT

A specialized measurement model in social research is described to demonstrate that when applied, the data derived thereof have the same properties as the data employed in the physical sciences. The mathematical properties of physical measurement are described with a classic example and then it is demonstrated how the proposed latent trait models satisfy the same requirements.

Key words: psychometrics, objective measurement, latent trait models, Rasch model.

Scope and Focus

Statistically speaking, a model is an optimal combination of measured variables that predict an outcome variable. In this sense a mathematical model replicates an empirical process. In the disciplines that study human action, there has always been a quest to explain decision making processes. A variety of mathematical models has been proposed to explain and predict decision making. However, when empirical data are involved, the measurement facets of the processes are called into question due to the nature of the data gathering methods and more importantly; to the alleged lack of means to produce objective measures.

Today few would contend that there are aspects of human action that must be measured to explain and predict behavior. Quantitative approaches are well established and justified for research in psychology, sociology, economics, education, ecology, health sciences and bioethics among other disciplines. In fact, psychometrics as a field is exclusively concerned with the measurement of human attributes. (Nunnally, J. & Bernstein, I. 1994). This paper does not argue in favor of a justification of quantitative methods in social sciences. What it does argue for is a particular approach to measurement that is fundamental to establish objectivity, reliability and validity of the data derived in social contexts from instruments and measurement processes.

While quantitative methods are routinely employed in social research; what is not widely known or accepted is that there is a measurement paradigm that produces psychometric data with properties equivalent to the data in the physical sciences. This paradigm is currently known as Latent Trait Modelling (LTM) and its foundations were -partly but decisively- developed by the Danish mathematician George Rasch between 1953 and 1960. (Rasch, G. 1960).

The Rasch Metric Model (RMM), produces linear measures from qualitative data. (Linacre, J. M. 2011, p.33). Decision making judgments made by respondents upon answering items on instruments designed to assess psychological constructs such as: cognitive ability or subject matter knowledge, are all in principle qualitative judgments. Attitudinal and perception traits such as opinion, preferences and values among others can also be measured by means of ordered category items and modelled with extensions developed of the Rasch Model for dichotomous responses.

Measurement properties

In the physical sciences objects are fundamentally measurable because the numerical properties of order and addition have physical referents (Embretson, S. Reise S. 2000, pp. 141-142). G. Rasch employed a well-known physical model to establish the mathematical relationships that hold in psychological measurement with a theoretical construct latent trait level as referent:

$$\boldsymbol{Acceleration} = \boldsymbol{Force}/\boldsymbol{Mass}$$

Taking the natural logarithm of both sides of the equation leads to a direct additive relationship:

$$\boldsymbol{log}(\boldsymbol{Acc}) = \boldsymbol{log}(\boldsymbol{Force}) - \boldsymbol{log}(\boldsymbol{Mass})$$

Analogously, when measuring cognitive or behavioral traits:

$$\boldsymbol{Task} = {}^{\boldsymbol{Latent\ Trait}}\!/_{\boldsymbol{Difficulty}}$$

This can also be transformed by:

$$\boldsymbol{log(Item\ Response) = log(b) - log(d)}$$

$$\boldsymbol{log\left(\frac{P_{si}}{(1 - P_{si})}\right) = b_s}$$

Where $\boldsymbol{P_{si}}$ is the number of items correctly answered or endorsed by subject s on an instrument (questionnaire, inventory) and $\boldsymbol{b_s}$ is the measure of standing on the trait of the person. The log transformation sets a common metric unit for the person trait.

$$\boldsymbol{log\left(\frac{(1 - P_{sk})}{P_{sk}}\right) = d_k}$$

Where $\boldsymbol{P_{sk}}$ is the number of persons failing or not endorsing an item on an instrument (questionnaire, inventory) and, $\boldsymbol{d_k}$ is the

$$\boldsymbol{p_s(x_i = 1 \mid \beta_s, \delta_i) = \frac{e^{(\beta_s - \delta_i)}}{1 + e^{(\beta_s - \delta_i)}}}$$

$$\mathbf{i = 1 \ldots .. k}$$

For each item a probabilistic expectation is established as follows:

$$\boldsymbol{p_s(x_i = 1 \mid \beta_s, \delta_i) = \frac{1}{1 + e^{(\beta_s - \delta_i)}}}$$

$$\mathbf{i = 1 \ldots .. k}$$

The resulting regression logistic model optimally combines the variables trait - task β_s δ_i, to predict the outcome. The model´s predictions are then subject to a number of rigorous tests and quality controls to ascertain objectivity, reliability and validity of the measurement instruments and processes. The mathematical properties of logistic regression ensure that when the resulting data fit the modelled expectations the measures become independent of the sample of persons and the sample of items (tasks) employed. (Hambleton, R.K. , Swaminathan, H. & Rogers, H. 1991). Another advantage of this method is that the precision of measures can be obtained for each individual facet, person or task. This is done by producing standard errors that are unique to each facet. This feature had not been available

Where $\boldsymbol{log(\ b)}$ is the person´s trait level and $\boldsymbol{log(d)}$ is the item's property and both are derived from a probabilistic function applying logistic regression as follows:

measure of difficulty of the cognitive or behavioral tasks. The log transformation sets a common metric unit for the person's trait and the item's property.

Generalizing the above over a number of S persons and I items on any developed psychometric instrument, Rasch´s Metric Model (RMM) establishes a probability of correct response or endorsements for each person and each item -denominated facets- in a measurement process.

For each person a probabilistic expectation is established as follows:

until the development of the LTM family of models.

Given the above, direct and precise comparisons of the persons standing on the trait can be made for any two (or more) persons:

$$\boldsymbol{\frac{P_{1i}/(1 - P_{1i})}{P_{2i}/(1 - P_{2i})} = \frac{\beta_1/\delta_i}{\beta_2/\delta_i} = \frac{\beta_1}{\beta_2}}$$

The resulting trait level comparisons have decisive practical, legal and ethical implications given the substantive decisions that can be made on the basis of measurement data derived under the RMM. Most importantly, the description demonstrates that the same requirements of order and addition can be met by data obtained in a psychometric process. When invariant meaning of measurement scores for human traits is desired, the Rasch Model is the appropriate choice. Rasch's mathematical model has been extended to be employed with ordered category data also known as Likert Scales (Andrich, D. 1978).

An Illustration of Latent Trait Modeling

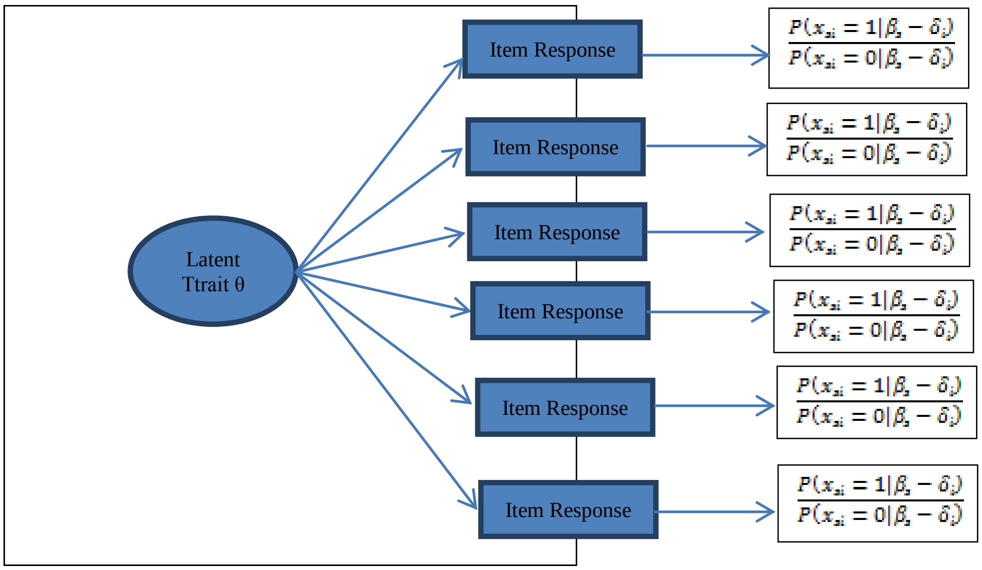

Figure 1

Implementation and Perspectives

The Rasch Metric Model is to date the most parsimonious and most explanatory of all the LTM alternatives. Technical resources for implementation are common in the field. An interesting and powerful resource is the LTM package in the R Statistical programming environment (Rizopoluos, D. 2006). A complete implementation of the Rasch family of models is provided by WINSTEPS
® (Linacre, J.M. 1991-2011). This contribution to the ISINI 2011 methodological panel recognizes that not all social research programmes require quantitative methods. The contention Is that when and if quantitative data are required , LTM techniques -particularly the Rasch Model- should be employed to ascertain the metric properties of the data.

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